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ФІЛОСОФІЯ НАУКИ І ТЕХНІКИ

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MEASUREMENT OF VAGUE PREDICATES

Purpose. The way of assignment of exact numerical truth value to any vague predicate sentence remains to be problematic. **Methodology.** I would like to propose one of the possible ways of estimation for vague sentences: to exploit the supevaluationists' idea of precisification for the interpretation of verity. We can think of the verity of a borderline sentence (the degree to which it is close to definite truth) as the proportion of permissible precisifications on which it is true. **Scientific novelty.** The proposed construal of degrees, interpreting verities on the basis of a measure over admissible precisifications, allows discrimination among borderline cases that would otherwise (on the standard supervaluationist account) all inhabit the same truth-value gap. My view of vague expressions assumes also that a borderline sentence may affect verity of another borderline sentence. The notion of relative verity reflects an intuitive assumption about possible semantic connections between applications of vague predicates. So-called forcing connection is a non-symmetric, transitive relation, which does not express any temporal or causal dependence between borderline cases; rather, it expresses a logical or semantic relation. I consider different kinds of forcing connection between vague expressions. **Conclusion.** Using of the notions “relative verity” and “forcing” provides preservation for borderline sentences of certain logical connections, which are postulated by classical logic.

Keywords: vagueness, borderline sentence, relative verity, super valuation.

Introduction

Vagueness usually is connected with predicate expressions such as ‘**x** is clever’, ‘**x** is tall’, ‘**x** is tired’, ‘**x** is bald’, ‘**x** is a child’, ‘**x** is a heap’, and so on. The distinguishing characteristic of such expressions is the seeming impossibility of drawing any sharp boundary between what the expression applies to and what it does not apply to.

Vague expressions characteristically have *borderline cases*, which are neither definitely true nor definitely false, and which thus belong to this grey zone. However, it is not correct that any vague predicate sentence **F(x)** divides the universe into three parts in a unique way, because the boundaries between these parts also are not sharp.

I share the degree-theoretical position that the truth function can assign extreme values **0** and **1** corresponding to definitely false and definitely true sentences, as well as fractional values to borderline cases. So, I am assuming that truth can proceed by degrees. To avoid any confusion of terms, I follow D. Edgington [1] in using the term "verity" for a degree of truth that is a measure of the closeness of the sentence to definite truth. I use "$\boldsymbol{v(\alpha)}$" ("verity of $\boldsymbol{\alpha}$") instead of "the degree of truth of the sentence $\boldsymbol{\alpha}$."

Vague predicates can be classified in a rather natural way on the basis of how their applicability might be measured. Degree-theoretic analysis seems most suitable for vague predicates that may be thought of as *measurable*—such as ‘tall’, ‘red’, and ‘child’. Such terms are usually connected with a corresponding numerical scale that provides some measure for the predicate’s applicability—e.g., ‘child’ and ‘adult’ are connected with measures of age; ‘tall’ and ‘short’ are connected with measures of height. Predicates that are measurable in this way are those for which it is most natural to use a numerical scale when speaking of their verity (their degree of truth).

Many vague predicates, such as ‘heap’ and ‘bald’, are measurable along more than one dimension. Not only quantity, but also the configuration of grains, as well as the density of their distribution, are relevant in determining whether a set of grains constitutes a heap. Likewise, baldness depends not only on the amount of hair, but also on its thickness, and distribution. The measure corresponding to such expressions seems to depend on more than one factor. The numerical estimation of such predicates is more sophisticated, although possible.

The application of the degree-theoretical approach seems even more difficult in the case of seemingly unmeasurable expressions, such as ‘beautiful’, ‘clever’, and ‘wicked’, which are not straightforwardly connected with any measure at all. Sometimes, for the sake of effective comparison, we use some conventional numerical scales, such as scores in a beauty contest or results of IQ testing. Of course, the expression 'she is wicked to the degree 0.43' seems very strange and unnatural. On the other hand, degrees of wickedness seem to be ordered linearly—for any individuals ***a*** and ***b*** from the relevant domain of discourse, either ***a*** is more wicked than ***b***, or ***b*** is more wicked than ***a***, or ***a*** and ***b*** are wicked to the same degree. Nevertheless, it is very hard to say to what degree ***a***, say, is more wicked than ***b***.

Problem statement

Thus, I suggest that any sentence containing a vague predicate expression can have numerical verity, regardless of the type of predicate. However, the most problematic thing is a method of valuation. Now, I would like to propose one of the possible ways of estimation for vague sentences.

The main text

The popular approach to vagueness known as 'supervaluationism' (K. Fine [2], H. Kamp [3], D. Lewis [6] and others) uses the idea of a permissible sharpening of a vague predicate. If we think of vagueness as a matter of semantic indecision, we can always stipulate a sharp boundary wherever one is lacking. Fine [2] uses the term 'precisification' for 'a legitimate way of making the language more precise'. One may treat a vague predicate **F** as precise by adopting (in a given situation) an arbitrary precise border between **F** and **F**, and we may then consider the whole range of permissible precisifications of this sort. In Lewis's general semantics (Lewis [6]) any sentence is characterized by *indices*—a sequence of coordinates, for time, place, possible world and so on—on which the truth-value of a sentence might depend and which make a precisification of the sentence permissible. These coordinates may include, for any one-dimensional predicate, some *delineation coordinate*—a real number that is taken as a precise cutoff point of the predicate (e.g. the boundary height for 'tall', the boundary age for 'child' etc.). For multi-dimensional predicates we consider a sequence of boundary-specifying numbers. Any precisification divides the universe sharply into the predicate's extension and its complement, in one of a set of permissible ways. For any permissible precisification of the predicate **F** and any individual **a**, either **Fa** is true or **~ Fa** is, but not both (the law of excluded middle holds on any particular precisification).

I would like to exploit the supevaluationists' idea of precisification for the interpretation of verity. A sentence **Fa** is definitely true (***v*(Fa) =1**)—'supertrue'—if it is true on all permissible precisifications, definitely false (***v*(Fa) =0**)—'superfalse'—if it is false on all permissible precisifications, and borderline if it is true on some but not all permissible precisifications. On the standard supervaluationist approach borderline sentences, being neither supertrue nor superfalse, yield a *truth-value gap* (the principle of bivalence does not hold). However, at least two supervaluationists (Lewis and Kamp) admit degree valuations for borderline sentences. Building on this idea, I would like to fill the truth-value gap by assigning each borderline sentence a degree of truth (verity), so that borderline sentences, too, count as bearers of truth. A wholesale precisification can be thought of as a monotonic non-decreasing mapping of the interval **[0,1]** to the classical pair **{0,1}**. We can think of the verity of a borderline sentence (the degree to which it is close to definite truth) as the proportion of permissible precisifications on which it is true. If μ is the totality of all permissible precisifications of **Fa**, and η is the number of precisifications on which **Fa** comes out true, then ***v*(Fa)** $= \eta/\mu$. A sentence that is true on relatively many precisifications will have a high degree of verity; a sentence that is true on relatively few precisifications will have a low degree of verity; and a sentence that is true and false on the same number of precisifications will have verity of **0.5**.

Thus, my approach combines elements of supervaluationism with the degree approach. D. Edgington [1] and also R. Keefe [4] consider such a combination a useful heuristic device. Although neither of them agrees entirely with such an account, each for different reasons, their approaches may be considered mutually complementary rather than contradictory. The proposed construal of degrees, interpreting verities on the basis of a measure over admissible precisifications, allows discrimination among borderline cases that would otherwise (on the standard supervaluationist account) all inhabit the same truth-value gap.

The notion of *relative verity* reflects an intuitive assumption about possible semantic connections between applications of vague predicates. To explain this idea I will use the following example (by T. Williamson [8, 154–155]). If Adam was born before Eve, then Eve is younger than Adam, so the sentence 'Eve is young' intuitively seems to be *no less true* than the sentence 'Adam is young'. Both sentences may be equally true or false (in definite cases). However, the simultaneous assertion that Adam is young and Eve is not young is intuitively inconsistent. At the same time, the sentences 'Adam is tall' and 'Eve is young' are semantically independent—all the following pairs of assertions are consistent: 'Adam is tall and Eve is young'; 'Adam is tall and Eve is not young'; 'Adam is not tall and Eve is young'; 'Adam is not tall and Eve is not young'.

I have defined the verity of a borderline sentence (the degree to which it is close to definite truth) as the proportion of permissible precisifications on which it is true. Any precisification divides the universe sharply into the predicate's extension and the extension of its complement, in one of a set of permissible ways. In other words, any precisification is a permissible way of assuming each borderline sentence has definite value (0 or 1). K. Fine [2, 126] associates precisifications with the range of *penumbral connections* ('penumbra' in B. Russell's sense [7, 149] of a grey zone). For instance, assuming 'Adam is young', we are committed to the following sentences: 'Adam is not old'; 'Eve (who is younger than Adam) is also young'; 'If Adam is young then Eve is young'; and so on. The relation between two such applications of a vague predicate remains constant, not depending on how one might choose to set a sharp border for the predicate. The generalization 'Anyone younger than a young man is not old' will be supertrue. The generalization 'Everybody is young and also not-young (old)' will be superfalse, because a contradiction is admissible under no precisification.

Following these ideas, my view of vague expressions assumes that a borderline sentence may affect verity of another borderline sentence. For any sentence **α**, the verity ***v*(α)** is the degree of closeness of the sentence **α** to definite truth. Every borderline sentence **α** (such that **0 < *v*(α) < 1**), considered in relation to another borderline sentence **β**, may also be characterized by a relative verity ***v*(α** relative to **β**), which is the value that **α** would have under the assumption that **β** is (definitely) true (***v*(β)** =1). We can ask what value we would be obliged to assign the borderline sentence **α** if we were to call the related borderline sentence **β** definitely true (cf. R. Keefe [4, 99]). The relative verity ***v*(α relative to β)** may be defined as the proportion of precisifications where both **α** and **β** are true relative to all the permissible precisifications where **β** is true. Furthermore, I would like to coin the meta-linguistic expression **β ⇒ α** ('**β forces α**') as an abbreviation for: ***v*(α relative to β) = 1** ('assigning the value **1** to **β** forces assigning the value **1** to **α**'); and two sentences are *forcing-connected* if one forces the other. Assume **Ya** means 'Adam is young' and **Ye** means 'Eve is young'. If both sentences are borderline cases (**0 < *v*(Ya) < 1** and **0 < *v*(Ye) < 1**), then the assumption that Adam is definitely young (***v*(Ya) =1**) *forces* the result that Eve is definitely young as well: (***v*(Ye)=1**). Thus, ***v*(Ye relative to Ya)** =**1**. Thus, the assumption that Adam is young shifts, as it were, the border of the predicate 'young', thereby changing the verity of all related borderline sentences.

We can understand the meta-linguistic expression **β ⇒ α** ('**β forces α**') as: 'any precisification that makes the sentence **β** true also makes the sentence **α** true'. Of course, only related sentences may be forcing-connected, but not all related sentences are thus connected. For instance, the assumption that 'Eve is young' forces neither 'Adam is young', nor 'Adam is not young', although these statements are related. The forcing connection is a non-symmetric, transitive relation, which does not express any temporal or causal dependence between borderline cases; rather, it expresses a logical or semantic relation. One can suppose that if **β** logically entails **α**, then **β ⇒ α**. However, if both **α** and **β** are simple (atomic) predicate sentences, we cannot speak of logical entailment, but only of *semantic entailment*.

Fine distinguishes between *internal* penumbral connections—between borderline cases of the same predicate (for instance, 'Adam is young' and 'Eve is young')—and *external* ones between borderline cases of different predicates (for

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instance, 'Adam is young' and 'Adam is old') [3, 129–130]. We can consider different kinds of forcing connection between vague expressions.

The expression '$\boldsymbol{\beta \Rightarrow \alpha}$' may be used for expressing *comparisons*, such as 'taller than', 'younger than' and in general 'more **F** than'. Assume that John is taller than Bill: $\boldsymbol{v(Tj) > v(Tb)}$. Indeed, the assumption that Bill is tall ($\boldsymbol{v(Tb) = 1}$) forces the conclusion that John, too, is tall: $\mathbf{Tb \Rightarrow Tj}$.

If the extension of the predicate **F** *analytically* includes the extension of the predicate **Q**, then corresponding simple predicate sentences are *semantically related*. Such is the case when one vague predicate *details* another, in the sense of adding one or more features to its intension. For example, the predicate 'ill with the flu' details the predicate 'ill' (although both predicates remain vague), so the extension of the latter includes the extension of the former. If **Fx** means '**x** is ill with the flu' and **Ix** means '**x** is ill', then for all states of affairs $\boldsymbol{v(Fx) \le v(Ix)}$, and $\mathbf{Fx \Rightarrow Ix}$.

A second kind of *inclusive relation* between predicates is presented by predicates expressing various degrees of the same feature (for example, 'invalid' – 'ill', 'baby' – 'child', 'fat' – 'overweight'). One may represent corresponding pairs of simple predicate sentences using the same predicate symbol and say: 'Bill is very overweight' instead of 'Bill is fat'. If **Fx** means '**x** is fat' and **Sx** means '**x** is overweight', then, for all states of affairs, $\boldsymbol{v(Fx) \le v(Sx)}$ and $\mathbf{Fx \Rightarrow Sx}$.

If the extensions of the predicates **F** and **Q** are *analytically disjoint* (disjoint in virtue of their semantic contents), then the corresponding simple predicate sentences are semantically *mutually exclusive*: $\mathbf{Fx \Rightarrow {\sim}Qx}$ and $\mathbf{Qx \Rightarrow {\sim}Fx}$. Contradictory sentences are not only *mutually exclusive* (the overlap of corresponding intervals is empty), but also *jointly exhaustive* (the unit of the corresponding intervals is the entire interval $\mathbf{[0, 1]}$). The pair of contradictory sentences containing the same predicate symbol is the simple predicate sentence and its negation ('tall' and 'not tall').

Let's now consider pairs of simple contradictory sentences containing various predicate symbols ('healthy' – 'ill', 'guilty' – 'innocent', 'child' – 'adult', 'bald' – 'hirsute'). Given a naturally restricted domain (such as the class of persons), one of such sentences may be expressed by the negation of another: '**x** is healthy' is the same as '**x** is not ill' and likewise for 'ill' and 'not healthy'. Indeed, the pairs of such predicate sentences divide the universe into the same parts—the truth domain of 'ill' coincides with the falsity domain of 'healthy'; grey zones coincide too, except for the difference in the direction of the axes.

Borderline cases of contradictory predicates seem to be compatible: somebody may be a borderline case between 'healthy' and 'ill' (which actually is true of most people). However, the assumption that 'John is healthy' ('John is not ill') is true ($\boldsymbol{v(Hj) = v({\sim}Ij) = 1}$) entails the conclusion that 'John is ill' ('John is not healthy') is false ($\boldsymbol{v(Ij) = v({\sim}Hj) = 0}$), and *vice versa*. Note that such a relation has a place not only in this state of affairs, but also in all applications of these predicates:

for any **x**, $\mathbf{Hx \Rightarrow {\sim}Ix}$ and $\mathbf{{\sim}Hx \Rightarrow Ix}$.

For contrary simple sentences we can formulate two conditions:

C1. Any two contrary sentences **Fx** and **Qx** are mutually exclusive: for any **x**, $\mathbf{Fx \Rightarrow {\sim}Qx}$ and $\mathbf{Qx \Rightarrow {\sim}Fx}$.

C2. For the *exhaustive set* of contrary predicates $\mathbf{F_1,\dots,F_n}$, the set of applications of all these predicates to the same individual **x** is jointly exhaustive: for any **x**, $\boldsymbol{v(F_1x) + \dots + v(F_nx) = 1}$, **so** $\mathbf{{\sim}F_1x \Rightarrow (F_2x \lor \dots \lor F_nx)}$.

For example, the simple predicate sentences '**x** is tall', '**x** is of average height', and '**x** is short' are contraries. This set of contrary sentences divides the universe into five successive parts: "definitely short", "borderline short-average", "definitely average", borderline average-tall", "definitely tall". The assumption that John is definitely short means that John is definitely not average, and so too the contrapositive: $\mathbf{Sj \Rightarrow {\sim}Aj}$ and $\mathbf{Aj \Rightarrow {\sim}Sj}$. However, the converse is not correct: for any **x**,

$\mathbf{{\sim}Ax \Rightarrow (Tx \lor Sx)}$; $\mathbf{{\sim}Sx \Rightarrow (Ax \lor Tx)}$, and $\mathbf{{\sim}Tx \Rightarrow (Ax \lor Sx)}$.

Applications of other measurable contrary predicates can divide the universe into more than three parts. For example, the set of predicates characterizing age can include the following nine successive zones (five definite and four borderline): 'definitely a child', 'borderline between a child and an adolescent', 'definitely an adolescent', 'borderline between an adolescent and a young adult', 'definitely a young adult', 'borderline between a young adult and middle-aged', 'definitely middle-aged', 'borderline between middle-aged and elderly', and 'definitely elderly'. Note that this number is arbitrary and depends on the purposes at hand.

Conclusion

Thus, any pair of contradictory sentences (borderline ones as well as definite ones) and any exhaustive set of contrary sentences are represented as mutually exclusive and also jointly exhaustive. Indeed, such connections between contradictory and contrary sentences are classical. Using of the notions "relative verity" and "forcing" provides preservation of certain classical logical connections also for borderline sentences.

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ВИМІРЮВАНІСТЬ РОЗПЛИВЧАСТИХ ПРЕДИКАТІВ

Мета. Спосіб приписування точного істиннісного значення кожному розпливчастому реченню залишається проблематичним. **Методологія.** Один із можливих засобів оцінки істинності розпливчастих речень засновано на використанні супер-оціночної теорії уточнень для інтерпретації істинності. Ступінь істинності розпливчастого речення (ступінь його наближення до визначеної істини) може бути представлена як пропорція можливих уточнень, при яких це речення є істинним. **Наукова новизна.** Визначення істинності, яке пропонується автором та інтерпретує ступінь істинності як міру допустимих уточнень, дозволяє провести градацію граничних речень, котрі при стандартному супер-оціночному підході попадають у так звану істиннісно-значущу шпарину. Підхід автора до розпливчастих виразів уможливлює також, що одне пограничне речення може впливати на ступінь істинності іншого такого речення. Поняття відносного ступеня істинності відображує інтуїтивне уявлення щодо можливих семантичних зв'язків між різними додатками розпливчастих предикатів. Так зване примусове відношення – це несиметричне транзитивне відношення, яке відображує не тимчасову або причинну залежність між граничними реченнями, а їх логічний або семантичний взаємозв'язок. Розглянуто можливі види відношень між розпливчастими реченнями. **Висновки.** Використання понять відносного ступеня істинності та примусового відношення дозволяють зберігати визначені логічні взаємозв'язки між граничними реченнями, обумовлені класичною логікою.

Ключові слова: розпливчастість, пограничне речення, відносний ступінь істинності, супероцінки.

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ИЗМЕРИМОСТЬ РАСПЛЫВЧАТЫХ ПРЕДИКАТОВ

Цель. Решение проблематичности способа приписывания точного истинностного значения каждому расплывчатому предложению. **Методология.** Один из возможных способов оценки истинности расплывчатых предложений основан на использовании супер-оценочной теории уточнений для интерпретации истинности. Степень истинности расплывчатого предложения (степень его приближения к определенной истине) может быть представлена как пропорция возможных уточнений, при которых это предложение истинно. **Научная новизна.** Предлагаемое определение истинности, интерпретирующее степень истинности как меру допустимых уточнений, позволяет провести градацию пограничных предложений, которые при стандартном супер-оценочном подходе попадают в так называемую истинностно-значную щель. Подход автора к расплывчатым выражениям допускает также, что одно пограничное предложение может оказывать влияние на степень истинности другого такого предложения. Понятие относительной степени истинности отражает интуитивное представление о возможных семантических связях между различными приложениями расплывчатых предикатов. Так называемое вынуждающее отношение – это несимметричное транзитивное отношение, которое выражает не временную или причинную зависимость между пограничными предложениями, а их логическую или семантическую взаимосвязь. Рассмотрены возможные виды вынуждающих отношений между расплывчатыми предложениями. **Выводы.** Использование понятий относительной степени истинности и вынуждающего отношения позволяют сохранить определенные логические взаимосвязи между пограничными предложениями, обусловленные классической логикой.

Ключевые слова: расплывчатость, пограничное предложение, относительная степень истинности, супер-оценки.

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